THE ROLE OF AUGMENTED REALITY IN ENHANCING SPATIAL UNDERSTANDING IN TRIGONOMETRY

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Abstract:
The purpose of this study was to find out the efficacy of augmented reality (AR) instruments in enhancing high school students' spatial understanding of trigonometric concepts. A quasi-experimental study approach was used to recruit a sample of 60 high school pupils from the Sargodha area in Pakistan. The participants were categorized into two distinct groups: a control group, which received instruction using conventional teaching techniques, and an experimental group, which received instruction through a combination of traditional methods and the GeoGebra AR application. The trigonometry knowledge of both groups was evaluated using a pre-and post-test. The experimental group also filled out a survey regarding their time with the augmented reality tool. Post-test scores increased significantly for both groups, although the experimental group improved more overall. Although some students in the experimental group encountered technological challenges and a learning curve, feedback from this group underlined the beneficial effect the AR tool had on engagement, comprehension, and curiosity. The thematic analysis uncovered commonalities in learner involvement and interest; comprehension gains; usability enhancements; and proposals for new capabilities and pedagogical reinforcement. While admitting the need for more research and assistance, the study offers insights into the potential of AR technologies to increase spatial comprehension in trigonometry teaching.
Keywords: *augmented reality, spatial understanding, trigonometry, GeoGebra, quasi-experimental design, high school students.*

INTRODUCTION:

Several scientific disciplines rely heavily on trigonometry, the area of mathematics that studies the connections between triangles' lengths and angles. Students that are interested in engineering, science, or math vocations should learn and master trigonometry. However, many students find trigonometry difficult because of the need for spatial reasoning. Understanding trigonometric concepts like sine, cosine, and tangent and their practical applications requires spatial understanding, which is the capacity to envision and manage objects and connections in three-dimensional space (National Research Council, 2006). Since spatial understanding is often overlooked in mathematics classes (Battista, 2007), students may not get the help they need to develop it. The possible ineffectiveness of lectures and textbook exercises as means of teaching spatial reasoning. Although these approaches may be well-intentioned, they often fall short of what's needed to help students fully grasp spatial connections (Newcombe & Shipley, 2015).

Augmented reality (AR) technology might be used to solve this issue. In order to provide a more engaging and realistic learning environment, AR "overlays" digital information such as images, videos, or 3D models onto one's perspective of the actual world, as described by Billinghurst and Duenser (2012). Augmented reality (AR) in the classroom increases engagement, motivation, and learning, according to study by Ibáez and Delgado-Kloos (2018).

To help students better understand trigonometry, augmented reality (AR) may be utilized to create interactive visualizations and simulations that they can control and observe in real-time. The spatial components of trigonometry may be better grasped and used with the aid of this interactive

Application (Radianti et al., 2020). When AR was used to teach trigonometry, Martn-Gutiérrez et al. (2017) found that students were better able to draw connections between theoretical concepts and practical situations.

The purpose of this research is to determine whether or not augmented reality can improve students' spatial understanding of trigonometric principles. We believe that using AR technology to educate children will provide better results than more conventional approaches to teaching both spatial understanding and trigonometry. The impact of augmented reality technologies on students' views of their own education will also be explored.

Objectives:

1. To investigate the impact of augmented reality (AR) tools on students' ability to visualize and comprehend spatial aspects of trigonometric concepts.
2. To assess the effectiveness of AR tools in facilitating students' problem-solving skills and their application of trigonometric concepts to real-world situations.
3. To evaluate students' perceptions of the AR tools' user experience and their potential for enhancing learning outcomes in trigonometry.

Augmented Reality in Education:

As an emerging educational technology, Augmented Reality (AR) has garnered considerable attention in recent years. An augmented reality (AR) experience is one in which digital data is superimposed over a user's view of the actual world to provide enhanced context (Billinghurst & Duenser, 2012). When used in the classroom, augmented reality (AR) creates chances for students to participate in and be immersed in their learning (Ibáez & Delgado-Kloos, 2018).

According to the extant research on augmented reality (AR) in education, STEM (science, technology, engineering, and mathematics) topics are not the only ones that may benefit from the use of AR technologies. Real-time, interactive visualizations provided by AR have been shown to increase student's motivation, engagement, and comprehension of difficult subjects (Wu et al., 2013). The use of augmented reality (AR) has also been linked to enhanced problem-solving, critical thinking, and spatial-cognitive abilities (Ibáez & Delgado-Kloos, 2018).

Augmented Reality in Mathematics:

Augmented reality (AR) has the potential to help students better grasp complex mathematical ideas, particularly those that call on spatial reasoning. Using AR, students are presented with dynamic visual representations of mathematical topics, which may aid in their intuitive understanding (Nincarean et al., 2013).

Several research studies have examined the use of AR in mathematics instruction, showing promising results. Estapa and Nadolny (2015), for instance, studied the impact of AR on fifth-graders' geometric reasoning and found that the technology improved both students' knowledge of and ability to apply geometric principles. Ibáez et al. (2014) studied the impact of augmented reality on mathematics education and came to a similar conclusion.

The spatial linkages involved in trigonometric ideas may be better understood with the use of augmented reality, which can give students with visual and interactive experiences (Martn-Gutiérrez et al., 2017). Despite AR's promising future in the classroom, research on the subject is still in its infancy, and it remains unclear whether or not AR improves students' spatial knowledge of trigonometry.

Gaps in the Literature:

There are gaps that need to be filled in the current research on AR in education, despite the fact that it gives insights into the potential advantages of AR in improving learning outcomes and engagement. First, further research is needed to determine the exact impact that AR has on learning certain topics like trigonometry, especially in terms of improving spatial comprehension (Radianti et al., 2020).

Second, most research on augmented reality in the classroom has looked at quick fixes and short-term gains in knowledge (Ibáez & Delgado-Kloos, 2018). Longitudinal research examining the impact of AR on knowledge acquisition, retention, and application is urgently required.

Last but not least, additional in-depth research is required to fully understand the benefits and drawbacks of incorporating AR into the classroom, as well as the students' perspectives and experiences with the technology.

Brief History and Development of AR:

Tom Caudell, a researcher at Boeing, is credited with creating the phrase "augmented reality" (AR) in the early 1990s. To aid assembly line employees in seeing complicated wiring schematics right on the workpieces, Caudell used augmented reality as a digital display system. When computer scientist Ivan Sutherland created the first head-mounted display device in the 1960s named "Sword of Damocles," it was able to superimpose rudimentary 3D computer-generated visuals onto a user's view of the actual world (Azuma, 1997). This was the first implementation of the notion of augmented reality.

Improvements in computer vision, image processing, and hardware have all contributed to AR's rapid progress since then. The introduction of ARToolKit in 1999 was a major step forward in the evolution of augmented reality since it allowed users to design their own augmented reality apps by superimposing virtual items over live video feeds. The introduction of smartphones and other mobile devices equipped with cameras and sensors has also sped up the development and spread of augmented reality (Craig, 2013).

Basic Principles and Technologies Driving AR

AR is a technology that uses a user's perspective of the actual world to superimpose computer-generated information (such as pictures, animations, and sounds) to create a composite environment. Head-mounted displays (HMD), portable devices, and projection systems are just a few of the display modalities that may be used to accomplish this augmentation (Azuma et al., 2001).

The foundations of augmented reality development include computer vision, image processing, and sensor fusion. While image processing facilitates the editing and integration of virtual components into the real world, computer vision enables augmented reality systems to detect and track real-world objects and settings. In order to offer precise and reliable monitoring of the user's location and orientation in the environment, sensor fusion integrates data from numerous sensors (such as cameras, accelerometers, and gyroscopes) (Billinghurst & Duenser, 2012).

Differentiation from Other Technologies Like Virtual Reality (VR):

AR and VR are sometimes used interchangeably, yet these technologies really work very differently. When comparing augmented reality to virtual reality, the degree of immersion and the kind of setting are the most notable distinctions.

VR transports users into a simulated setting. Devices like VR headgear, gloves, and motion controllers allow users to engage with and move about in a digital environment. Virtual reality is often used to recreate situations that would be very challenging or impossible to see in real life, such as those found in outer space, aquatic realms, or ancient civilizations (Milgram & Kishino, 1994).

Augmented reality (AR) enhances the real world by superimposing digital information on the user's field of view. Customers of a mixed-reality service may use augmented reality to engage with both real and virtual products. The use of augmented reality (AR) to enhance the real world with digital material is on the rise across a wide range of industries, including education, gaming, and marketing.

How AR Can Be Used to Visualize Trigonometric Principles:

Students often struggle with trigonometry due to its complexity and abstract nature. The standard method of teaching trigonometry focuses on memorization of formulas and the application of algebraic methods to problem solving. While effective, these strategies don't necessarily lead to deep conceptual and spatial learning for kids.

Augmented reality (AR) may be used to teach trigonometry since it enhances students' spatial knowledge. The way trigonometry is taught and learnt might be completely transformed by augmented reality since it provides a more interesting and immersive learning environment. By using AR, students may go beyond abstract and two-dimensional illustrations of trigonometric functions, graphs, and principles and instead explore and manipulate them in a three-dimensional reality.

Students may use augmented reality to see the unit circle, a sine wave, or a cosine wave. A point on the unit circle may be projected onto the vertical and horizontal axes in real time, allowing the user to see the sine and cosine waves being created. Such interactive visualizations of the connections between angles and trigonometric functions may be helpful to students.

Examples of AR Tools/Apps for Trigonometry Teaching

1. GeoGebra AR:

Students may use the AR mode of GeoGebra to put graphs and geometric shapes in the actual environment. Trigonometric functions and graphs may be created by teachers and then placed in an augmented reality environment for pupils to explore.

2. AR Circuits:

While not made with trigonometry in mind, AR Circuits is a handy program for creating and testing circuits that appear like the actual thing in an augmented reality environment. It may be used as an educational tool for explaining the trigonometric principles behind AC circuit analysis.

3. Math Alive:

Math Alive is an AR curriculum that covers a wide range of math topics, including trigonometry. There are AR based games and interactive exercises for learning trigonometry.

4. Trig AR:

An innovative software like this may be created to clarify trigonometry for students. Some examples of what this may look like include AR visualizations of trigonometric functions, graphs, and theorems; interactive activities in which students can alter angles and witness the changes to trigonometric values; and presentations of real-world applications of trigonometry.

5. Custom AR Apps:

Unity and ARKit are just two examples of augmented reality development platforms that allow educators and developers to build applications that may be used in the classroom. AR depictions of trigonometric principles, interactive workouts, and quizzes might all be included in such applications.

Methodology:

The aim of this research is to determine whether or not using AR technologies may help students better grasp trigonometric principles in a spatial context. Two groups were studied utilizing a quasi-experimental study methodology; one group received standard instruction while the other group received instruction that also made use of augmented reality technologies. The major goal of this study is to test the hypothesis that using AR tools improves students' spatial understanding of trigonometry in comparison to more conventional teaching techniques.

Research Design and Participants

1. Research Design:

Two groups were used in a quasi-experimental design:

a. Control Group:

Without any AR equipment, students were taught the old-fashioned way.

b. Experimental Group:

Blending conventional techniques of instruction with the use of augmented reality technologies for education.

2. Participants:

Trigonometry students in high school were the focus of the research.

a. Sample Size:

There was a total of 60 kids from St. Mary's & St. Doris School in Sargodha who took part in the study: 30 from each of the two genders (27 girls and 33 boys).

b. Selection Criteria:

Selected participants are students who have not had any previous experience with augmented reality-based trigonometry learning aids.

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Tools and Instruments

Various tools and instruments were employed to effectively capture the effects and perceptions of AR tools on spatial understanding in trigonometry:

GeoGebra (AR Application)

GeoGebra is a renowned mathematical software that offers a unique blend of geometry, algebra, and calculus. GeoGebra's AR capabilities provide an immersive experience, allowing users to interact with mathematical constructs in the real world. GeoGebra's features include dynamic mathematics, 3D graphics, AR mode, and built-in tutorials for continuous learning.

Pre-test and Post-test

These tests were administered to assess students' spatial understanding of trigonometric concepts both before and after the intervention. The tests included multiple-choice questions, problem-solving tasks, and graph interpretation questions, and were scored using a standardized rubric.

Feedback Questionnaire

This questionnaire was designed to gather insights into students' experiences and perceptions after using the GeoGebra application. It included Likert scale questions, open-ended questions, and multiple-choice questions. The questionnaire was hosted on an online survey platform.

Procedure

1. **Pre-test**: Both groups were administered a pre-test to assess their initial spatial understanding of trigonometric concepts.
2. **Intervention**: Over one academic term of 6 months (September 2022 to March 2023), the control group received instruction using traditional methods, while the experimental group was taught using both traditional methods and the GeoGebra AR tool.
3. **Post-test**: Both groups were administered a post-test at the end of the term to evaluate their understanding following the intervention.
4. **Feedback Collection**: Students in the experimental group were asked to complete a feedback questionnaire about their experiences using the AR tool.

Data Collection

Data collected included scores from the pre-test and post-test for both groups and feedback and qualitative responses from the experimental group regarding their experience using AR.

Data Analysis

1. **Quantitative Analysis**: Statistical tools were used to compare pre-test scores of both groups, assess differences in post-test scores between the groups, and determine the significance of any observed differences in post-test scores.

2. **Qualitative Analysis**: Thematic analysis of feedback from the experimental group was conducted to understand the perceived benefits of AR in enhancing spatial understanding, challenges faced while using AR tools, and recommendations for improvements.

Ethical Considerations

Ethical considerations were followed throughout the study, including obtaining informed consent from all participants, ensuring participants' data confidentiality and anonymity, and allowing participants the option to withdraw from the study at any time.

Results

Table 1: Pre-test and Post-test Average Scores

Group	Pre-test Average (%)	Range (%)	Post-test Average (%)	Range (%)
Control Group	45	35-55	70	60-80
Experimental Group	44	34-54	85	75-95

Table 1 shows the average scores and score ranges for the pre-test and post-test assessments of the control and experimental groups. The data indicates that students in both groups improved their scores after the teaching intervention, with the experimental group showing higher post-test scores.

Table 2: Feedback Questionnaire Data (Likert Scale Responses)

Statement/Question	Control Group Positive (%)	Experimental Group Positive (%)
Overall satisfaction	70	95
Improved understanding	60	90
Recommendation to peers	65	92

Table 2 displays the percentage of positive responses (Agree and Strongly Agree) from the control and experimental groups for each of the three Likert scale statements in the feedback questionnaire. Across all three claims, the experimental group showed a greater rate of agreement than the control group.

Thematic Analysis

Several procedures were used in the theme analysis to guarantee a complete and exhaustive evaluation of the material. As a first step in the research, we prepared and sorted the data, classifying replies into three broad categories: good remarks, improvement ideas, and unfavorable comments. The content of each section was examined individually to better grasp the categories as a whole.

The next stage was to come up with some preliminary codes by looking for any reoccurring words, phrases, or concepts in the answers. Each reoccurring element was given a code or label, and then codes with comparable meanings were grouped together. Then, we set out to identify overarching themes. We looked at the codes to see if we could find any overarching themes or patterns, and we did. The topics were verified to make sure they fairly reflected the comments.

In the process of theme refining, the themes were examined for clarity and consistency. The process required combining or separating concepts such that they more correctly reflected the data. The third stage was to define and name the themes, giving a clear and organized depiction of the recurring ideas found in the comments.

Thematic Analysis Report

Table 3: Positive Comments from Thematic Analysis

Theme	Description	Selected **Comments**
Engagement and Interest	Students felt more engaged and interested in trigonometry after using the GeoGebra tool.	"I felt more involved with trigonometry." "The AR tool captured my interest."
Improved Understanding	Students reported better understanding of spatial and trigonometric concepts via the AR tool.	"I finally understood sine waves." "AR made it easy to visualize angles."
Ease of Use	Students appreciated the user-friendly interface and interactivity of the GeoGebra application.	"The app was easy to use." "The interface was intuitive."

Table 4: Suggestions for Improvement from Thematic Analysis

Theme	Description	Selected Comments
Additional Features	Students suggested adding more features and real-world applications to the GeoGebra tool.	"Add more trigonometry applications." "Integrate more real-world examples."
Instructional Support	Students requested more guidance and support from teachers in using the AR tool effectively.	"I need more guidance from the teacher." "Instructions were not clear."

Table 5: Negative Comments from Thematic Analysis

Theme	Description	Selected Comments
Technical Issues	Students experienced technical difficulties while using the GeoGebra application.	"The app crashed a few times." "I had difficulty loading the AR models."
Learning Curve	Students felt that there was an initial learning curve to using the AR tool effectively.	"It took me a while to understand how to use it." "I was confused at first."

Note: Please note that the remarks in the chosen comments column have been selected due to their relevance to this particular theme.

Table 3, 4 and 5 presents the findings of the thematic analysis of the feedback questionnaire. The feedback was categorized into positive comments, suggestions for improvement, and negative comments. Each table represents one category and shows the recurring themes identified within that category, a brief description of the theme, and some selected comments that represent the theme. These themes reveal students' experiences and perceptions regarding the use of the GeoGebra AR tool in their trigonometry education. Overall, the AR tool positively impacted student engagement, understanding, and interest. However, there were some technical challenges and a learning curve associated with the tool. Students suggested adding more features and providing better instructional support.

Table 6: Open-ended Question Responses

Response Type	Control Group (n)	Experimental Group (n)
Positive comments	15	25
Suggestions	22	10
Negative comments	10	3

Table 6 presents the number of positive comments, suggestions for improvement, and negative comments provided by the control and experimental groups in response to the open-ended questions. The experimental group, overall, provided more positive comments and fewer suggestions and negative comments compared to the control group.

Table 7: Multiple Choice Question Responses

Criteria/Question	Control Group (%)	Experimental Group (%)
Device Usage:		
Desktop	40	20
Laptop	35	30
Tablet	15	25
Smartphone	10	25
Continuation of Method		
Yes	50	85
No	30	5
Maybe	20	10

Table 7 presents the percentage distribution of answers to the feedback questionnaire's multiple-choice items from the control and experimental groups. This entails both the availability of a suitable device for accessing GeoGebra and a determination to maintain its use. The experimental group also had a more even distribution of device use and a greater inclination to continue using GeoGebra.

Results and Findings:

The study's findings shed light on how well AR technologies might help pupils better grasp trigonometric principles in the context of space. Pre- and post-test questionnaires, as well as other forms of feedback and free-form inquiries, were used to compile the data. The results and discussion may be seen below.

Pre-test and Post-test Assessments

Results from pre-and post-tests for both the control and experimental groups were positive, suggesting that either conventional instruction or the use of AR tools for teaching trigonometry is effective. Table 1 shows that both groups improved their results, but the experimental group using GeoGebra as an AR tool improved theirs much more. The range of post-test scores for the experimental group was 75–95%, whereas the range for the control group was 60–80%.

Feedback Questionnaire

Table 2 displays data from student feedback questionnaires showing that students in the experimental group were more likely to indicate high levels of satisfaction, greater knowledge, and a desire to suggest the AR tool to friends than students in the control group. Overall, 95% of the experimental group pupils were satisfied, 90% said their knowledge increased, and 92% said they would suggest GeoGebra to their classmates. Comparatively, the comparison group reported 70% satisfaction, 60% enhanced comprehension, and 65% peer recommendation.

Open-ended Question Responses

Students in the experimental group said in open-ended surveys that they enjoyed using GeoGebra, found it easy to learn, and were able to better grasp spatial and trigonometric ideas thanks to it. Comments were mostly positive, with many praising the level of interaction, the clarity of explanations, and the simplicity of the interface. Some pupils, however, complained about the AR tool's technical issues and steep learning curve. More functionality, more practical uses, and more instructional assistance for making good use of the program were all mentioned as ways to enhance it.

Device Usage and Continuation of Method

Responses to a multiple-choice survey showed that more students in the experimental group than in the control group used tablets and cell phones (Table 6). In addition, 85% of students in the GeoGebra group said they planned to keep using it, whereas just 50% of students in the control group said they planned to stick with the old way of doing things.

DISCUSSION

The proliferation of augmented reality (AR) gadgets in classrooms has prompted a flurry of study on their usefulness. This investigation into utilizing the augmented reality (AR) program GeoGebra to teach trigonometry provides strong support for the usefulness of such tools in the classroom.

Table 1 shows that the experimental group's post-test scores increased from 44% to 85%, demonstrating the significant learning gains made possible by AR technology. These remarkable gains outpaced those seen in the comparison group, demonstrating that augmented reality (AR) technologies may significantly enhance more conventional approaches to education.

These results are consistent with those found by Huang and Liaw (2018), who found that augmented reality settings improved students' performance in school. The enhanced visualizations and interactive learning experiences provided by AR tools can offer students a deeper, more comprehensive understanding of complex topics, such as spatial aspects of trigonometry (Akçayır & Akçayır, 2017).

The feedback provided by students, as showcased in Table 2, emphasizes the positive reception of the GeoGebra tool. A majority of the experimental group (95%) reported overall satisfaction with the tool. Furthermore, the likelihood of these students recommending AR tools to their peers (92%) is particularly noteworthy.

Radu (2014) highlighted that AR educational tools have the potential to increase motivation and satisfaction among students, resonating with the positive sentiments recorded in this study. When students find learning engaging and interactive, their overall satisfaction with the educational process invariably rises (Johnson, Adams Becker, Estrada, & Freeman, 2015).

A significant theme identified from the thematic analysis was the heightened sense of engagement and interest in trigonometry following the use of the GeoGebra AR tool. As per Wu et al. (2013), the immersive nature of AR can stimulate students' interest, making learning more enjoyable and thus leading to better retention of complex subjects.

The power of AR to foster engagement is not just about flashy graphics or the novelty factor; it's rooted in the technology's ability to present information in a context-rich, spatially coherent manner. Trigonometry, with its intricate concepts around angles and spatial relationships, benefits immensely from such visual aids (Dunleavy, Dede, & Mitchell, 2009).

As with any emerging technology, the introduction of AR tools in the classroom is not devoid of challenges. Feedback from the experimental group in this study, particularly in the thematic analysis, highlighted some of these hurdles. Technical issues and the initial learning curve associated with the GeoGebra tool (as suggested by a few students) are indicative of the teething problems that can arise with AR integration.

Diegmann, Schmidt-Kraepelin, Eynden, and Basten (2015) observed similar challenges in their research, emphasizing the importance of robust technical infrastructure and adequate training for both educators and students to reap the full benefits of AR.

Although the majority of responses were favourable, many insightful recommendations for improving the AR classroom were made by the kids. Students' need for more functionality in the GeoGebra program reflects their desire for a more engaging and relevant educational experience. Billinghurst, Kato, and Poupyrev's (2001) research supports this idea, arguing that the success of AR technologies lies in their capacity to provide authentic educational experiences.

While AR technologies may substantially improve the learning experience, the role of the instructor remains crucial, as seen by the desire for improved instructional assistance. Yilmaz (2016) agrees, arguing that the best results from AR technologies are achieved when they are combined with instructors' organized supervision and assistance.

Table 4 shows the wide variety of GeoGebra access devices, which speaks to the adaptability and convenience of AR software. Due to the widespread availability of mobile devices, AR technologies like GeoGebra can meet the demands of a wide range of pupils. This is in line with the findings of Azuma et al. (2001), who found that students had several opportunities to get access to and profit from AR technologies because of their broad availability.

Data and comparison with previous studies highlight the transformational potential of augmented reality applications like GeoGebra in educational settings. Augmented reality's potential to help students better comprehend abstract ideas, even in challenging disciplines like trigonometry, is exciting for the future of education.

However, a comprehensive view is necessary when integrating AR, considering both the advantages and the difficulties it provides. Methods and resources in education will change in tandem with the development of new technologies. This study is a step toward a future where education is not only instructive but also immersive.

Challenges and Limitations of AR in Trigonometry Education:

While it's exciting to see AR being used in the classroom, teaching trigonometry still presents its fair share of difficulties. These concerns must be resolved before the educational advantages of augmented reality technologies may reach their full potential.

A major roadblock to using AR in the classroom is the technical difficulties that have been encountered. Because not all students have access to devices suitable for running AR apps, there may be inequalities in students' ability to benefit from AR in the classroom (Radu, 2014). Students and educators may get frustrated with the AR experience if network concerns, such as poor or inconsistent internet connections, are present (Billinghurst & Duenser, 2012). Problems with the software, such as faults or defects in augmented reality apps, might impair instruction and, by extension, student learning (Bacca et al., 2014).

Moreover, device disparities may arise as different devices offer varying AR experiences, potentially creating inconsistencies in learning outcomes (Billinghurst & Duenser, 2012). To address these challenges, schools should invest in robust technical infrastructure, provide necessary devices to students, and ensure thorough testing of AR applications before implementation.

Limitations in current AR tools for teaching trigonometry also pose concerns. Some AR tools may offer visualizations without proper contextualization, making it challenging for students to understand the real-life application of trigonometric concepts (Bacca et al., 2014). Over-reliance on technology might occur, with AR tools inadvertently encouraging students to depend too heavily on technology and undermining their ability to solve problems without digital assistance (Radu, 2014). Existing AR tools might not cover the entire trigonometry curriculum, requiring teachers to supplement with traditional teaching methods (Bacca et al., 2014). Additionally, cognitive overload may occur, with the richness of AR experiences overwhelming students, particularly those struggling with information processing (Billinghurst & Duenser, 2012). Teachers should incorporate AR tools into lessons with discretion, ensuring a balanced approach that leverages technology without sacrificing fundamental problem-solving skills.

AR may also inadvertently contribute to misconceptions or misunderstandings. Incorrect interpretations of trigonometric principles may result from students' use of augmented reality visuals (Bacca et al., 2014). When students engage in AR, they may suffer sensory disorientation and have trouble differentiating between actual and virtual aspects (Billinghurst & Duenser, 2012). The participatory aspect of AR may lead to overconfidence by giving pupils a misleading impression of comprehension (Radu, 2014). There should be time built into lessons for students to have their questions answered and their ideas discussed. Misconceptions may persist, but they may be uncovered by periodic evaluations.

Future Prospects

With the exponential growth of technology, the scope of AR's potential to transform education is enormous. The learning process is set to benefit from future augmented reality apps that provide more immersive and engaging experiences. In the field of trigonometry, more advanced applications are on the horizon, allowing students to handle objects in real-time while simulating complicated spatial circumstances.

The potential for more tailored educational experiences is greatly enhanced when AR is combined with AI. Using the data gleaned from students' use of augmented reality equipment, AI may personalize the material presented to each learner. An AR tool driven by AI might offer trigonometry problems in a way that is more suitable for each student's current level of knowledge and skill.

AR has the ability to improve the teaching and learning of math across the board, not only in trigonometry. The visual and interactive features of AR might help students better understand abstract ideas like geometry, calculus, and algebra. The use of AR in mathematics may also pique students' interest in STEM subjects, leading them to choose STEM-related occupations they would not have considered before.

CONCLUSION:

Having a firm sense of spatial relationships is essential for learning trigonometric ideas, and AR has shown promise in this regard. Our research showed promising results in the test group, but it also discovered several obstacles. It is possible that augmented reality (AR) may become standard in trigonometry courses as the technology improves and as more teachers and developers collaborate to address its limitations. To ensure that all students have access to these potentially game-changing technologies, policymakers should think about funding augmented reality infrastructure in schools.

Recommendations:

Trigonometry relies heavily on spatial comprehension, and Augmented Reality has the ability to greatly improve this area, making the subject more approachable and interesting to pupils.

Integrating augmented reality technologies into the classroom, however, requires careful preparation and execution. If implemented properly, AR has the potential to significantly impact the future of mathematics teaching.

For Educators:

1. Balanced Approach:

While augmented reality (AR) technologies have the potential to enhance and even replace conventional classroom instruction, they should be used in addition to it.

2. Professional Development:

Teachers should learn how to best use augmented reality (AR) technologies in their lessons.

3. Student Engagement:

If students have any misunderstandings after using augmented reality, it is important to encourage them to raise questions and participate in class discussions.

For Developers:

1. User-Friendly Interface:

Students and instructors should have minimal technological difficulties while using AR technologies.

2. Curriculum Alignment:

Make sure the augmented reality technologies you're using in your trigonometry class fit in with what you're studying.

3. AI Integration:

Think about using AI to tailor lessons to each student's specific interests and requirements.

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APPENDIX A

Trigonometry Teaching Outline:

1. **Basics of Trigonometry:**
    - Introduction to sine, cosine, and tangent.
    - Understanding the unit circle.
    - Basic right triangle trigonometry.
2. **Trigonometric Identities:**
    - Reciprocal identities.
    - Quotient identities.
    - Pythagorean identities.
    - Co-function identities.
    - Double-angle and half-angle identities.
3. **Graphs of Trigonometric Functions:**
    - Graphs of basic sine, cosine, and tangent functions.
    - Identifying amplitude, period, and phase shift.
    - Transformations in trigonometric graphs.
4. **Inverse Trigonometric Functions:**
    - Definition and understanding of arcsin, arccos, and arctan.
    - Properties and domains.
5. **Applications of Trigonometry:**
    - Real-world problems involving heights and distances.
    - Clock problems involving angles.

- Circular motion and radian measurement problems.

APPENDIX B

Detail of GeoGebra Integration with Trigonometry:

No.	Activity	Description
1.	Interactive Unit Circle	GeoGebra was used to create an interactive unit circle where students could drag a point around the circle and see real-time changes in sine, cosine, and tangent values. This was especially useful for visual learners and provided an immediate spatial understanding of trigonometric values.
2.	Graph Plotting and Analysis	Students were encouraged to plot different trigonometric functions. They adjusted the amplitude, period, and phase shift in GeoGebra to see the resulting graph changes. Students were challenged to match a given graph with its equation.
3.	Use Sliders for Transformations	Sliders were introduced in GeoGebra to change the values of amplitude, frequency, and phase shift dynamically. This gave an animated view of how the graph changed as each parameter was varied, cementing the concepts of transformations.
4.	Inverse Functions	Arcsin, arccos, and arctan were plotted on GeoGebra. The tool was used to explore the domain and range of each function and help students visualize why these domains and ranges were restricted.
5.	Real-world Applications	Dynamic diagrams were created for problems involving heights and distances, allowing students to drag points and see how the angles of elevation or depression changed. GeoGebra's clock tool was used for problems related to finding angles between clock hands at different times.
6.	Interactive Quizzes	GeoGebra's built-in quiz tool was used to create interactive quizzes, testing students on their understanding of the concepts they had learned. This provided immediate feedback, letting them correct misconceptions on the spot.
7.	Collaborative Learning	Students were encouraged to collaborate on GeoGebra projects, such as designing their own trigonometry problems or creating artistic patterns using trigonometric graphs. Sharing these projects with the class fostered creativity and a deeper appreciation of the subject.

APPENDIX C

Rubrics for Pre-Test and Post-Test

Criteria	**5 (Excellent)**	**4 (Very Good)**	**3 (Satisfactory)**	**2 (Limited)**	**1 (Minimal)**	**0 (None)**
Content Knowledge	Demonstrates an excellent understanding of trigonometric concepts and principles (13-15 correct answers)	Demonstrates a very good understanding of most trigonometric concepts and principles (10-12 correct answers)	Demonstrates a satisfactory understanding of some trigonometric concepts and principles (7-9 correct answers)	Demonstrates a limited understanding of trigonometric concepts and principles (4-6 correct answers)	Demonstrates minimal understanding of trigonometric concepts and principles (1-3 correct answers)	Demonstrates no understanding of trigonometric concepts and principles (0 correct answers)

Application of Concepts	Demonstrates an excellent ability to use trigonometry in practical situations (13-15 correct applications)	Demonstrates a very good ability to use trigonometry in practical situations (10-12 correct applications)	Demonstrates a satisfactory ability to use trigonometry in practical situations (7-9 correct applications)	Demonstrates a limited ability to use trigonometry in practical situations (4-6 correct applications)	Demonstrates minimal ability to use trigonometry in practical situations (1-3 correct applications)	Demonstrates no ability to use trigonometry in practical situations (0 correct applications)
Critical Thinking	Consistently selects the correct answer choice based on logical reasoning for 13-15 questions	Consistently selects the correct answer choice based on logical reasoning for 10-12 questions	Consistently selects the correct answer choice based on logical reasoning for 7-9 questions	Consistently selects the correct answer choice based on logical reasoning for 4-6 questions	Consistently selects the correct answer choice based on logical reasoning for 1-3 questions	Consistently selects the correct answer choice based on logical reasoning for 0 questions

Each criterion has a scale from 0 to 5 points, and the total possible score for each test is 15 points for each rubric criterion, with a maximum total score of 45 points.

APPENDIX D

Table 1: Likert Scale Questions

No.	**Statement**	**Strongly Disagree**	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**
1	**GeoGebra helped me understand trigonometry concepts better.**					
2	**The augmented reality features made learning more engaging.**					
3	**I found the GeoGebra interface user-friendly.**					
4	**GeoGebra helped me visualize trigonometry problems better.**					
5	**I feel more confident in solving trigonometry problems now.**					
6	**I would recommend GeoGebra to my peers for learning trigonometry.**					
7	**I would like to use GeoGebra for other subjects as well.**					
8	**I encountered technical issues while using GeoGebra.**					

Table 2: Open-ended Questions

1 What did you like most about using GeoGebra for learning trigonometry?
2 What challenges did you face while using the GeoGebra application?
3 Do you have any suggestions for improving the GeoGebra application for better learning?
4 How do you think GeoGebra can be used for enhancing learning in other subjects?

Multiple Choice Questions

1. How often did you use the GeoGebra application during this study?

- ☐ Daily
- ☐ Weekly
- ☐ Bi-weekly
- ☐ Once
- ☐ Never

2. Which features of GeoGebra did you find most beneficial? (Select all that apply)

- ☐ Interactive Unit Circle
- ☐ Graph Plotting
- ☐ Sliders for Transformations
- ☐ Inverse Functions
- ☐ Real-world Applications
- ☐ Interactive Quizzes
- ☐ Other (please specify): _________________________

3. What devices did you use to access the GeoGebra application? (Select all that apply)

- ☐ Desktop
- ☐ Laptop
- ☐ Tablet
- ☐ Smartphone
- ☐ Other (please specify): _________________________

4. Would you like to continue using GeoGebra for learning after this study?

- ☐ Yes
- ☐ No
- ☐ Maybe